Modeling the Behavior of RLC Circuits Using FPAA

Emil Dimitrov Manolov, Filip Todorov Koparanov and Mihail Hristov Tzanov

Abstract – **The paper presents an approach for implementation of FPAA prototypes that model the behavior of RLC circuits. To this aim, the analog modeling method is used. The standard procedure for obtaining the block diagram of the model from the dynamic equation is described. This diagram is used to build the FPAA electronic analog of the studied circuit. The frequency responses of two passive RLC circuits are examined in this way. The measured results are compared with theoretical calculations of the values of the amplitude ratios of the studied circuits for different frequencies. The comparison shows very good agreement between theory and practice and confirms the effectiveness and applicability of the methodology.**

Keywords – **Behavioral modeling, Dynamic equations, RLC circuits, Frequency-response, FPAA**

I. Introduction

The dynamic equations are a universal approach for modeling the behavior of the electrical circuits [1]. They are differential equations, which order depends on the number of the energy storage elements (capacitors and inductors) in the circuit [2].

The dynamic equations can be solved by using different classical analytical methods (substitution, operator, or state variable). Modern programs as Matlab [3], Matematica [4], Mathcad [5], ect. support these computational procedures.

In the contemporary engineering practice, the dynamic equations can be solved by applying analog modeling methods [1]. To this aim, the equations are transformed in block diagrams that can be studied by using Simulink [6]. In this way the solving method and the results are presented in graphical form that is very close to the engineering perception.

As it is presented in [7], the block diagrams, which model the second order differential equations of RLC circuits, can be practically implemented by using real analog computing components. These components are integrators and amplifiers and they are based on the standard operational amplifiers. This approach is used to implement KHN active filters.

The Field Programmable Analog Arrays (FPAAs) are a modern trend in analog electronics. They are programmable integrated circuits that implement analog functions. The programming changes component values and interconnections in the chip, so the function can be chosen dynamically while FPAAs are operating in a system.

E. D. Manolov, F. T. Koparanov, and M. H. Tzanov are with the Department of Electronics, Laboratory of Semiconductor Devices and Integrated circuits, Room 1437, Faculty of Electronic Engineering and Technologies, Technical University - Sofia, 8 Kliment Ohridski blvd., 1000 Sofia, Bulgaria, e-mails: edm@tu-sofia.bg, fkop@abv.bg, mzep@abv.bg

The FPAAs are very appropriate for fast prototyping of various analog and mixed signal circuits [8]. Nowadays the most popular FPAAs are the chips of Anadigm Inc [9].

The paper discusses the implementation of experimental electronic models that behave analogously with RLC electrical circuits. To this aim, a standard procedure for obtaining the block diagram of the model from the dynamic equation of the studied circuit is described. The diagram is used to build the FPAA electronic analog of the circuit. The frequency-responses of two passive RLC circuits are investigated in this way. The measured results are compared with theoretical calculations of the amplitude ratios of the studied circuits for different frequencies. The comparison shows very good agreement between the theory and the practice and confirms the effectiveness and the applicability of the methodology.

II. Procedure for Implementation and Investigation of FPAA Prototype

The procedure for implementation and investigation of the FPAA prototype, which models the behavior of the studied RLC circuit, will be presented through a simple example.

A. Studied circuit

Fig.1 shows a RC circuit that operates as high-frequency filter.

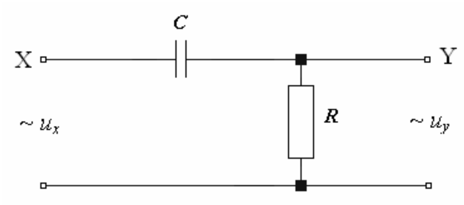

Fig.1. RC circuit

The goal is to build the FPAA electronic model, to examine the frequency-response and to estimate the correspondence between practical measurements and theoretical computations for this circuit.

B. Differential equation of the circuit

The differential equation of the examined circuit can be obtained by using the following expressions:

$$c\frac{du_c}{dt} = \frac{u_y}{R} \tag{1}$$

$$u_c = u_x - u_y \tag{2}$$

The substitution of Eq. (2) into Eq. (1) leads to:

$$\frac{du_y}{dt} + \frac{1}{RC}u_y = \frac{du_x}{dt} \tag{3}$$

After integration the Eq. (3) transforms to:

$$u_y = u_x - \frac{1}{RC}\int u_y dt \tag{4}$$

The Eq. 4 will be used to obtain the block diagram of the considered model.

C. Block diagram

The block diagram for solving the differential equation is shown on Fig.2. It is obtained according to Eq.(4) and consists of integrator *Int*, amplifier *Ampl* and summing block *Sum*. The cutoff frequency f_o of the filter can be controlled by changing the value of RC constant.

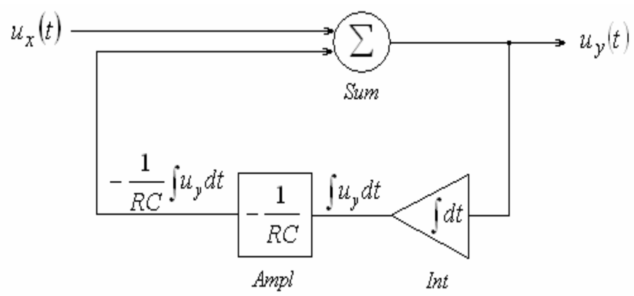

Fig.2. Block diagram of RC circuit

D. FPAA implementation of the block diagram

The FPAA model (Fig.3) corresponds to the developed block diagram. The small differences are due to the specifics of the available functional blocks in the library of the used AnadigmDesigner2 development program [9]. The *Gain_Inv_1* and the *Gain_Inv_2* are inverting amplifiers, which gain can be programmed between 0.01 and 100. The constant of the *Integrator* can vary between 0.04 and 10 [1/µs]. The *Sum_Inv* is an inverting amplifier with summing of the signals of the inputs. The clock frequency of the components of the circuit is 250 kHz, which ensures practically operation of the circuit up to 100 kHz.

E. Transfer function of the circuit

The frequency-response of the circuit can be determined by using voltage division expression:

$$\dot{A} = \frac{u_y}{u_x} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{1}{1 - j\frac{1}{\omega RC}} \tag{5}$$

The amplitude ratio of the frequency response is:

$$A = \frac{1}{\sqrt{1 + \left(\frac{1}{2\pi fRC}\right)^2}} \tag{6}$$

F. Practical examination of the FPAA model

For examination of the developed prototype, a sine-wave signal is applied to the input $u_x(t)$ of the circuit and the probe of the oscilloscope is connected with $u_y(t)$ output (see Fig.3). The frequency of the input signal is changed between $0.1f_o$ and $10f_o$. Fig.4 shows the results from examination of the FPAA prototype for three values of the RC constant: 0.001592 s, 0.0001592 s and 0.00001592 s. The first value corresponds to the cutoff frequency and equals to 100 Hz, the second – to 1000 Hz, and the third – to 10 kHz. The desired values of the RC constant are set by giving appropriate values of the gain of the *Gain_Inv_2* and the constant of the *Integrator*.

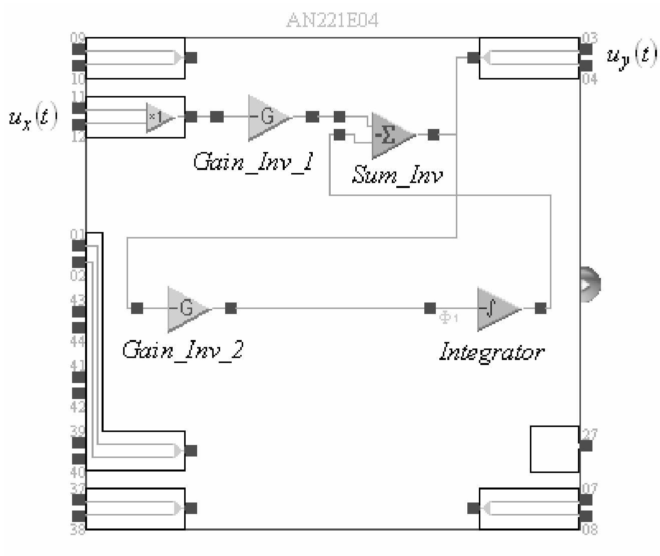

Fig.3. FPAA model

The smooth lines in Fig.4 present the amplitude ratio of the frequency response calculated according to Eq.(6). The left line is for cutoff frequency that equals to 100 Hz, the middle line is for cutoff frequency that equals to 1000 Hz, and the third line is for cutoff frequency that equals to 10 kHz. The different points around these lines present the experimental results for the amplitude ratio. These results are measured at the output of the FPAA model and normalized versus their maximum values. It is evident that calculated and measured values are very close.

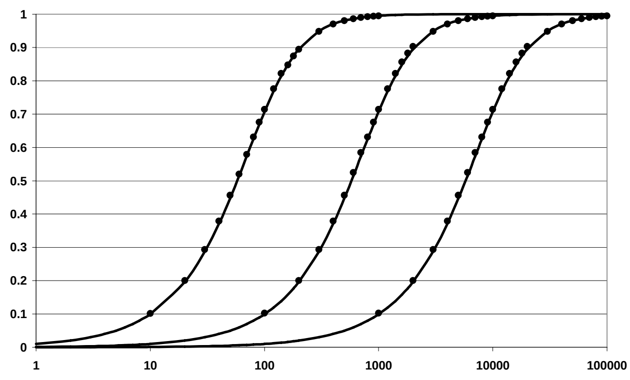

Fig. 4. Results from investigation the transfer function of the circuit

III. RLC CIRCUIT EXAMINATION

Fig.5 shows a series RCL circuit. The input signal is u_x and the output is u_y. The differential equation, which describes the circuit's behavior, is:

$$\frac{d^2u_y}{dt^2}+\frac{R}{L}\frac{du_y}{dt}+\frac{1}{CL}u_y=\frac{d^2u_x}{dt^2} \tag{7}$$

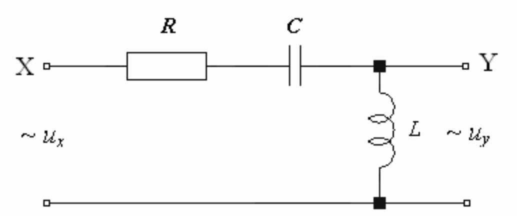

Fig.5. RLC circuit

The resonant radian frequency ω_o and the Q-factor are

$$\omega_o=2\pi f_o=\frac{1}{\sqrt{LC}} \tag{8}$$

and

$$Q=\omega_o\frac{L}{R} \tag{9}$$

Then the differential equation transforms to:

$$\frac{d^2u_y}{dt^2}+\frac{\omega_o}{Q}\frac{du_y}{dt}+\omega_o^2u_y=\frac{d^2u_x}{dt^2} \tag{10}$$

The block diagram for solving the Eq.(10) is shown on Fig.6. According to the figure the output signal is:

$$u_y=-\left(-u_x+\frac{\omega_o}{Q}\int u_y dt+\omega_o^2\iint u_y dt\right) \tag{11}$$

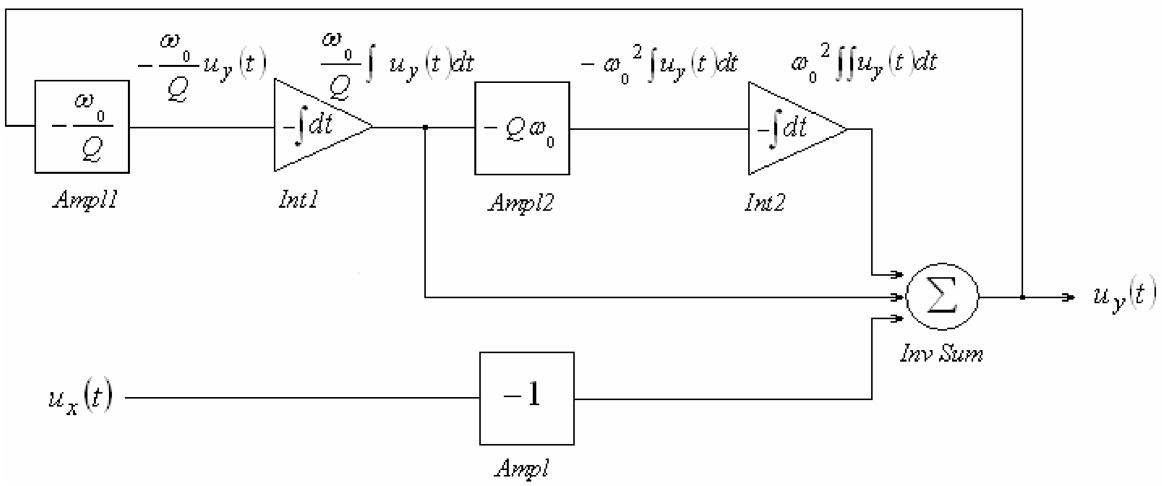

Fig.6. Block diagram of RLC circuit

Fig.7 presents the FPAA prototype of the discussed circuit. It is based on the block diagram on Fig. 6. The amplifying blocks *Ampl, Ampl1, and Ampl2* on Fig.6 are implemented by the corresponding inverting gain stages *Gain_Inv, Gain_Inv1, Gain_Inv2*. The integrators *Int1 and Int2* are implemented with *Inv_Int_1* and *Inv_Int_2* blocks on Fig. 7.

The results from investigation of the frequency response of the circuit for R=400Ω, C=100nF, and L=254mH are shown on Fig.8. To this aim, a sinusoidal exciting signal with sweep of the frequency between 100 Hz and 10 kHz is applied to the input $u_x(t)$. The output signal is read at the screen of the oscilloscope. The smoothed line presents the values at the output $u_y(t)$ that are computed by using the voltage division formula:

$$\dot{A}=\frac{u_y}{u_x}=\frac{1}{1-\frac{1}{\omega^2CL}-j\frac{R}{\omega L}} \tag{12}$$

$$A=\frac{1}{\sqrt{\left(1-\frac{1}{\omega^2CL}\right)^2+\left(\frac{R}{\omega L}\right)^2}} \tag{13}$$

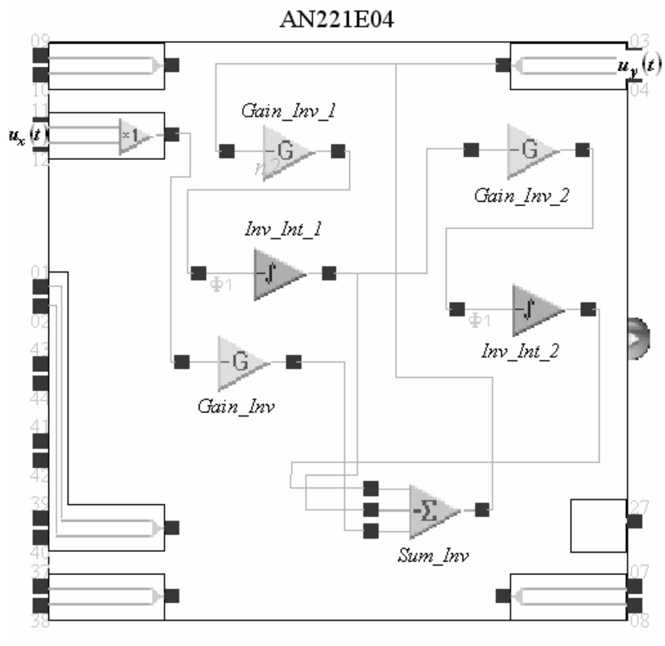

Fig.7. FPAA electronic analog of the RLC circuit

The points around smoothed line present the normalized measured results at the output of the FPAA prototype. The calculated and measured values are very close, again.

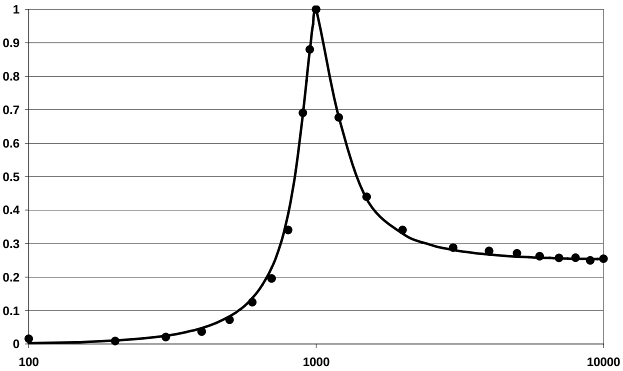

Fig.8. Results from investigation of RLC circuit

Fig.9 depicts the response of the circuit to the square wave signal, which is applied to the input $u_x(t)$. The results allow to investigate directly the behavior of the circuit by studying the output voltage.

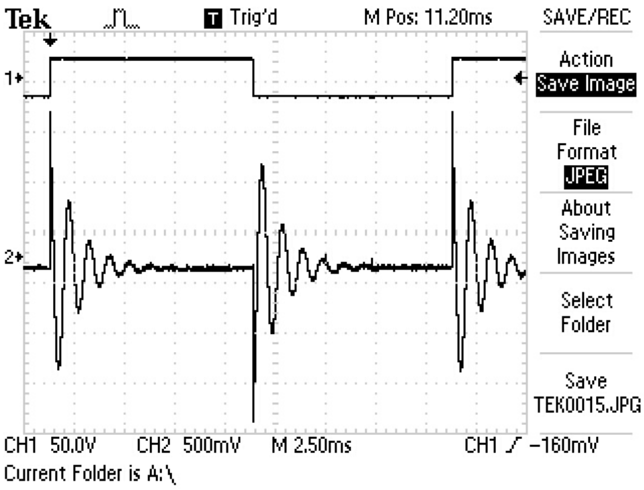

Fig.9. Response of the RLC circuit to the square wave exciting signal

Taking into consideration that

$$i = \frac{1}{L}\int u_y dt \tag{14}$$

the current through the series components can be also examined. It appears as a scaled function at the output of the inverting integrator *Inv_Int_1* (Fig.7).

In the same manner, by applying various excitation signals to the input, the behavior of the circuit can be examined in different conditions.

IV. CONCLUSION

The paper presents a methodology to build FPAA electronic models that imitate the behavior of passive RLC circuits. The methodology is based on the analog modeling principles. To this aim, firstly, the differential equation that describes the operation of the circuit should be obtained. After, this equation is used as a basis for development of the block diagram and the seeking FPAA prototype. The designed prototype can be applied for examination of the behavior of the modeled passive RLC circuit, or as its electronic equivalent.

Two types of circuits are investigated by using the described approach. The obtained results confirm empathically the presented methodology. It can be applied in research, engineering and educational practice.

V. ACKNOWLEDGEMENT

The research presented in this paper was funded by the Internal Competition of TU-Sofia - 2009, (Contract No 091ni032-03).

REFERENCES

[1] W. Siebert. Circuits, Signals, and Systems. MIT Press. 1986.
[2] R. Dorf, J. Svoboda. Introduction to Electric Circuits. John Wiley & sons Inc. 1999.
[3] http://www.mit.edu/people/abbe/matlab/ode.html
[4] http://www.math.armstrong.edu/faculty/hollis/mmade/
[5] http://www.che.wsu.edu/~rzollars/odesolvers.pdf
[6] S. Karris. Introduction to Simulink with Engineering Applications. Orchard Publications. 2006
[7] L. P. Huelsman. Ph. E. Allen. Introduction to the Theory and Design of Active Filters. McGraw-Hill, 1980.
[8] E. Manolov. *Research and Educational Experiments with Field-Programmable Analog Array.* Proceedings of the 12th International Conference Mixed Design of Integrated Circuits and Systems. MIXDES 2005, Kraków, Poland, pp. 975-980.
[9] http://www.anadigm.com